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On the absence of simultaneous reflection and transmission in integrable impurity systems

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Abstract

We establish that the Yang-Baxter equations in the presence of an impurity do in general not admit solutions of simultaneous transmission and reflection. Upon a mild assumption on the products in the impurity degrees of freedom, the only exceptions to this are diagonal bulk theories whose scattering matrices are constant phases, such as the free Boson and Fermion, the Federbush model and their generalizations. These anyonic solutions do not admit the possibility of excited impurity states.
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1 Introduction

Integrable quantum field theories in 1+1 space-time dimensions in the presence of a boundary have received a considerable amount of attention in recent years. One of the central aims is to find explicit solutions to the consistency equations in the presence of a boundary, which result as a consequence of factorizability, namely the Yang-Baxter equation [1, 2], the bootstrap equation [3] and also crossing [4, 5]. Explicit solutions are known for various theories, such as affine Toda field theories with real [6, 7, 8, 9] and purely imaginary coupling [10, 11], (in particular the sine-Gordon model [4, 12, 13, 14] and its supersymmetric version [15, 16]), the Gross-Neveu model [17], $N = 1$ [18] and $N = 2$ [19] supersymmetric theories, the nonlinear sigma models [20, 21, 22] and theories with infinite resonance states [23].

Part of the motivation for this great interest is coming from the fact that boundaries play a natural role in string theory. In the context of condensed matter physics, boundaries allow for instance the description of non-trivial constrictions in quantum wires. In order to understand realistic materials, it is in addition further important to investigate the effects of impurities (defects, inhomogeneities). For this latter situation much less is known at present. Besides the purely reflecting case, which is equivalent to the aforementioned boundary problem, there exist some solutions for purely transmitting impurities [24]. However, hitherto only few examples are known for the situation of simultaneously occurring reflection and transmission [25, 26, 27]. All these studied examples are related either to the free Fermion or Boson.

In [25] an argument was provided, which manifests that integrable parity invariant impurity systems with a diagonal bulk S-matrix, apart from $S = \pm 1$, do not allow simultaneously non-trivial reflection and transmission amplitudes. In this note we address the question, whether the set of possible bulk theories, which admit such a behaviour of the impurity, can be enlarged when the corresponding S-matrix is taken to be non-diagonal and parity is allowed to be broken. It will turn out that non-diagonal bulk scattering theories do not admit the possibility of simultaneous reflection and transmission on the defect when integrability of the theory is maintained. On the other hand, allowing parity breaking will slightly enlarge the set of bulk theories with such a behaviour of the defect. Also allowing additional degrees of freedom in the impurity does not increase the possible set of solutions. In fact, we demonstrate that, whenever reflection and transmission occur at the same time, the defect can only have one degree of freedom.

2 Defect Yang-Baxter equations

Integrability is, as usual in this context, identified with the factorization of the n-particle scattering matrix into two-particles ones. Many of the properties, these two-particle scattering matrices have to satisfy, result from the exploitation of the associativity of the so-called Faddeev-Zamolodchikov (FZ) algebra [28]. This also holds in the presence of a boundary [1, 2, 3] or an impurity [25], which formally can be associated to an element of the algebra with zero rapidity. We briefly want to recall this derivation, by following largely [25], with the difference that we also allow additional degrees of freedom in the inhomogeneity, corresponding to possible excited impurity states, and pay attention to parity. The latter means in particular, that amplitudes may be different when particles hit the defect from the left or from the right, a property known for instance in the context of lattice integrable models, see e.g., [29]. Indicating particle types by Latin and degrees of freedom of the impurity by Greek letters, the "braiding" relations of creation operators $Z_i(\theta)$ of a particle of type i with rapidity θ and defect operators Z_α in the state α can be written as

$$\begin{aligned} Z_i(\theta_1)Z_j(\theta_2) &= S_{ij}^{kl}(\theta_1-\theta_2)Z_k(\theta_2)Z_l(\theta_1), &(1)\\ Z_i(\theta)Z_\alpha &= R_{i\alpha}^{j\beta}(\theta)Z_j(-\theta)Z_\beta + T_{i\alpha}^{j\beta}(\theta)Z_\beta Z_j(\theta)\,, &(2)\\ Z_\alpha Z_i(\theta) &= \tilde{R}_{i\alpha}^{j\beta}(-\theta)Z_\beta Z_j(-\theta) + \tilde{T}_{i\alpha}^{j\beta}(-\theta)Z_j(\theta)Z_\beta. &(3)\end{aligned}$$

We employed Einstein's sum convention, that is we assume sums over doubly occurring indices. The left/right reflection and transmission amplitudes are denoted by $R/\tilde{R}$ and $T/\tilde{T}$, respectively. We suppress the explicit mentioning of the dependence of Z_α on the position in space and assume that it is included in α. For the treatment of a single defect this is not relevant anyhow, but it becomes of course important when considering multiple defects.

Using the relations (1)-(3) twice leads to the constraints

$$\begin{aligned} S_{ij}^{kl}(\theta)S_{kl}^{mn}(-\theta) &= \delta_i^m\delta_j^n, &(4)\\ R_{i\alpha}^{j\beta}(\theta)R_{j\beta}^{k\gamma}(-\theta) + T_{i\alpha}^{j\beta}(\theta)\tilde{T}_{j\beta}^{k\gamma}(-\theta) &= \delta_i^k\delta_\alpha^\gamma, &(5)\\ R_{i\alpha}^{j\beta}(\theta)T_{j\beta}^{k\gamma}(-\theta) + T_{i\alpha}^{j\beta}(\theta)\tilde{R}_{j\beta}^{k\gamma}(-\theta) &= 0\,. &(6)\end{aligned}$$

The same equations also hold after performing a parity transformation, that is for $R \leftrightarrow \tilde{R}$ and $T \leftrightarrow \tilde{T}$ in (5)-(6). Apart from the degrees of freedom in the impurity, these relations agree with those proposed in [24] and for $R = \tilde{R}$, $T = \tilde{T}$ with those in [25]. In the situation in which transmission is absent they reduce to expressions which may be found already in [1, 2].

The Yang-Baxter equations are derived as usual by exploiting the associativity of the ZF-algebra. Commencing with an initial state of the form $Z_i(\theta_1)Z_j(\theta_2)Z_\alpha$ and commuting in the order as depicted in figure 1 (the picture is to be read as equality, in the sense that the two scattering events are equal and the part in the middle of the defects serves as the income on the right and as the outcome for the left defect scattering process), we obtain the defect Yang-Baxter equations by reading off the coefficients from the linear independent asymptotic states of the form $Z_i(-\theta_1)Z_j(-\theta_2)Z_\alpha$, $Z_i(-\theta_1)Z_\alpha Z_j(\theta_2)$, $Z_i(-\theta_2)Z_\alpha Z_j(\theta_1)$ and $Z_\alpha Z_j(\theta_2)Z_i(\theta_1)$ as

$$\begin{aligned}
S_{ij}^{kl}(\theta_{12})R_{l\alpha}^{m\beta}(\theta_1)S_{km}^{np}(\hat{\theta}_{12})R_{p\beta}^{t\gamma}(\theta_2) &= R_{j\alpha}^{l\beta}(\theta_2)S_{il}^{mp}(\hat{\theta}_{12})R_{p\beta}^{k\gamma}(\theta_1)S_{mk}^{nt}(\theta_{12}), && (7)\\
S_{ij}^{kl}(\theta_{12})R_{l\alpha}^{m\beta}(\theta_1)S_{km}^{np}(\hat{\theta}_{12})T_{p\beta}^{t\gamma}(\theta_2) &= T_{j\alpha}^{t\beta}(\theta_2)R_{i\beta}^{n\gamma}(\theta_1), && (8)\\
S_{ij}^{kl}(\theta_{12})T_{l\alpha}^{m\beta}(\theta_1)R_{k\beta}^{p\gamma}(\theta_2) &= R_{j\alpha}^{l\beta}(\theta_2)S_{il}^{pn}(\hat{\theta}_{12})T_{n\beta}^{m\gamma}(\theta_1), && (9)\\
S_{ij}^{kl}(\theta_{12})T_{l\alpha}^{m\beta}(\theta_1)T_{k\beta}^{p\gamma}(\theta_2) &= T_{j\alpha}^{l\beta}(\theta_2)T_{i\beta}^{k\gamma}(\theta_1)S_{kl}^{pm}(\theta_{12}). && (10)
\end{aligned}$$

We abbreviated here the rapidity sum $\hat{\theta}_{12} = \theta_1 + \theta_2$ and difference $\theta_{12} = \theta_1 - \theta_2$. In the absence of degrees of freedom in the impurity, the first relation (7) was originally obtained in [1, 2], whereas (8)-(10), apart from a few obvious typos in the indices, were first derived in [25]. Note that when multiplying the equation (9) by $S(\theta_{21})$ from the left, it becomes identical to equation (8) upon using the unitarity relation (4) and a subsequent exchange of θ_1 and θ_2. Thus, we only need to treat three independent equations.

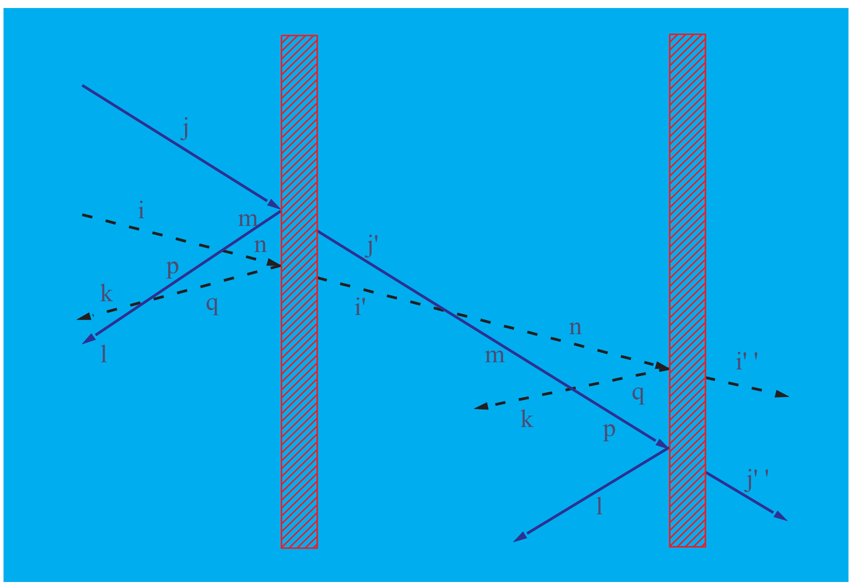

Figure 1: Defect Yang-Baxter equations.

Starting with an initial state in a different order leads to non-equivalent sets of equations. For instance taking $Z_\alpha Z_i(\theta_1)Z_j(\theta_2)$ as the initial state simply leads to the same equations as (7)-(10) with $R \leftrightarrow \tilde{R}$ and $T \leftrightarrow \tilde{T}$. Commencing on the other hand with $Z_i(\theta_1)Z_\alpha Z_j(\theta_2)$ and reading off the coefficients from the linear independent asymptotic states of the form $Z_i(-\theta_1)Z_\alpha Z_j(-\theta_2)$, $Z_i(-\theta_1)Z_j(\theta_2)Z_\alpha$, $Z_j(\theta_2)Z_\alpha Z_i(\theta_1)$ and $Z_\alpha Z_j(-\theta_2)Z_i(\theta_1)$ leads to

$$\begin{aligned}
R_{i\alpha}^{k\beta}(\theta_1)\tilde{R}_{j\beta}^{l\gamma}(\theta_2) &= R_{i\beta}^{k\gamma}(\theta_1)\tilde{R}_{j\alpha}^{l\beta}(\theta_2), && (11)\\
\tilde{R}_{l\beta}^{q\gamma}(\theta_2)S_{kj}^{lp}(\hat{\theta}_{12})T_{i\alpha}^{k\beta}(\theta_1) &= S_{lk}^{qp}(\theta_{12})T_{i\beta}^{l\gamma}(\theta_1)\tilde{R}_{j\alpha}^{k\beta}(\theta_2), && (12)
\end{aligned}$$

$$S^{st}_{lq}(\theta_{12})R^{q\gamma}_{p\beta}(\theta_1)S^{lp}_{ik}(\hat{\theta}_{12})\tilde{T}^{k\beta}_{j\alpha}(\theta_2) = R^{s\beta}_{i\alpha}(\theta_1)\tilde{T}^{t\gamma}_{j\beta}(\theta_2), \tag{13}$$
$$\tilde{T}^{q\gamma}_{l\beta}(\theta_2)S^{lp}_{kj}(\hat{\theta}_{12})T^{k\beta}_{i\alpha}(\theta_1) = T^{p\gamma}_{t\beta}(\theta_1)S^{qt}_{ik}(\hat{\theta}_{12})\tilde{T}^{k\beta}_{j\alpha}(\theta_2). \tag{14}$$

Clearly the three sets of Yang-Baxter equations (7)-(10), (7)-(10) with $R \leftrightarrow \tilde{R}$ and $T \leftrightarrow \tilde{T}$ and (11)-(14) are not equivalent. In the special case when $R = 0$ the equation (10) can be turned into (14) by means of the unitarity relation (5). On the other hand, when $T = 0$ the equations (11) remain a non-trivial requirement which links the left and right reflection amplitude via the impurity degrees of freedom.

In order to achieve a more concise formulation, let us re-write the defect Yang-Baxter equations in tensor form in the bulk indices. Employing the usual convention $(A \otimes B)^{kl}_{ij} = A^k_i B^l_j$ for the tensor product, the three non-equivalent equations in (7)-(10) take on the form

$$S(\theta_{12})[\mathbb{I} \otimes R^{\beta}_{\alpha}(\theta_1)]S(\hat{\theta}_{12})[\mathbb{I} \otimes R^{\gamma}_{\beta}(\theta_2)] = [\mathbb{I} \otimes R^{\beta}_{\alpha}(\theta_2)]S(\hat{\theta}_{12})[\mathbb{I} \otimes R^{\gamma}_{\beta}(\theta_1)]S(\theta_{12}), \tag{15}$$
$$S(\theta_{12})[\mathbb{I} \otimes R^{\beta}_{\alpha}(\theta_1)]S(\hat{\theta}_{12})[\mathbb{I} \otimes T^{\gamma}_{\beta}(\theta_2)] = R^{\gamma}_{\beta}(\theta_1) \otimes T^{\beta}_{\alpha}(\theta_2), \tag{16}$$
$$S(\theta_{12})[T^{\beta}_{\alpha}(\theta_2) \otimes T^{\gamma}_{\beta}(\theta_1)] = [T^{\beta}_{\alpha}(\theta_1) \otimes T^{\gamma}_{\beta}(\theta_2)]S(\theta_{12}), \tag{17}$$

whereas (11)-(14) can be equivalently written as

$$R^{\beta}_{\alpha}(\theta_1) \otimes \tilde{R}^{\gamma}_{\beta}(\theta_2) = R^{\gamma}_{\beta}(\theta_1) \otimes \tilde{R}^{\beta}_{\alpha}(\theta_2), \tag{18}$$
$$[T^{\beta}_{\alpha}(\theta_2) \otimes \mathbb{I}]S(\hat{\theta}_{12})[\tilde{R}^{\gamma}_{\beta}(\theta_1) \otimes \mathbb{I}]S(\theta_{12}) = T^{\gamma}_{\beta}(\theta_2) \otimes \tilde{R}^{\beta}_{\alpha}(\theta_1), \tag{19}$$
$$[\mathbb{I} \otimes \tilde{T}^{\beta}_{\alpha}(\theta_2)]S(\hat{\theta}_{12})[\mathbb{I} \otimes R^{\gamma}_{\beta}(\theta_1)]S(\theta_{12}) = R^{\beta}_{\alpha}(\theta_1) \otimes \tilde{T}^{\gamma}_{\beta}(\theta_2), \tag{20}$$
$$[T^{\beta}_{\alpha}(\theta_1) \otimes \mathbb{I}]S(\hat{\theta}_{12})[\tilde{T}^{\gamma}_{\beta}(\theta_2) \otimes \mathbb{I}] = [\mathbb{I} \otimes \tilde{T}^{\beta}_{\alpha}(\theta_2)]S(\hat{\theta}_{12})[\mathbb{I} \otimes T^{\gamma}_{\beta}(\theta_1)]. \tag{21}$$

Making now the assumption that the product in the impurity degrees of freedom is symmetric in the sense

$$A^{\beta}_{\alpha}(\theta_1) \otimes B^{\gamma}_{\beta}(\theta_2) = A^{\gamma}_{\beta}(\theta_1) \otimes B^{\beta}_{\alpha}(\theta_2) \qquad A, B \in \{R, T, \tilde{R}, \tilde{T}\} \tag{22}$$

we can for instance eliminate $T(\theta)$ in (16)*. Taking thereafter $\theta_2 = 0$ we obtain

$$S(\theta)\left[\mathbb{I} \otimes R^{\beta}_{\alpha}(\theta)\right] S(\theta) = R^{\beta}_{\alpha}(\theta) \otimes \mathbb{I}\ . \tag{23}$$

This means that the scattering matrix has to be proportional to the permutation operator $\mathbb{P}$

$$S(\theta) = \mathbb{P}\sigma\,, \tag{24}$$

where σ is a diagonal matrix with properties $\sigma_{ij}\sigma_{ji} = 1$. Besides the free Fermion ($\sigma_{ij} = \sigma_{ji} = -1$) and free Boson ($\sigma_{ij} = \sigma_{ji} = 1$) also the Federbush model [30] and the generalized coupled Federbush models [31] are solutions to (24).

*For $A = R$, $B = \tilde{R}$ and vice versa the assumption (22) is the same as (18). Therefore, only for the remaining choices of A, B the relations (22) are in fact assumptions.

Alternatively we can see this fact also by a similar argument as put forward originally in [25]. Considering from the start in equations (15)-(17) the diagonal case, that is setting $S_{ij}^{kl}(\theta) = S_{ij}(\theta)\delta_{il}\delta_{jk}$, $T_{i\alpha}^{j\beta}(\theta) = T_{i\alpha}(\theta)\delta_{ij}\delta_{\alpha\beta}$ and $R_{i\alpha}^{j\beta}(\theta) = R_{i\alpha}(\theta)\delta_{ij}\delta_{\alpha\beta}$, the equations constrain the bulk S-matrix as

$$S_{ab}(\theta_{12}) = S_{ab}(\hat{\theta}_{12}) \qquad \text{and} \qquad S_{ab}(\theta_{12})S_{ba}(\hat{\theta}_{12}) = 1\,. \tag{25}$$

It is clear that these equations demand the scattering matrix to be rapidity independent, but apart from the solutions of the free Boson and Fermion, i.e., $S = \pm 1$, suggested in [25], they also allow as solutions S-matrices of Federbush type as mentioned above. In that case they are of the form $S_{ab} = \exp(i\lambda_{ab})$, $S_{ba} = \exp(-i\lambda_{ab})$ with $\lambda_{ab} \in \mathbb{R}$. For this option to be possible, it is crucial to note that parity invariance is broken, such that in general $S_{ab} \neq S_{ba}$.

In summary: *Apart from rapidity dependent scattering matrices which are of the anyonic type (24), simultaneous reflection and transmission in an integrable system with an impurity is always absent, whenever the products in the impurity degrees of freedom satisfy (22).*

3 Defect bootstrap equations

Besides exploiting the associativity of the ZF-algebra in the above version, the presence of bound states in the bulk theory also leads to powerful constraints. Despite the fact that equation (24) already manifests that S is independent of the rapidity (likewise does (25)), let us exploit the associativity of the expression which reflects this situation

$$Z_a\left(\theta + i\eta_{ac}^b + i\varepsilon/2\right) Z_b\left(\theta - i\eta_{bc}^a - i\varepsilon/2\right) = i\Gamma_{ab}^c Z_c\left(\theta\right)/\varepsilon\,, \tag{26}$$

for $\varepsilon \to 0$. As conventional we denote here the three particle vertex on mass-shell by Γ_{ab}^c and the real fusing angles by η.

Commuting then in the manner as depicted in figure 2 leads to non-trivial constraints for the defect scattering matrices. This means scattering the particles a and b on the defect and fusing afterwards to particle c should be equivalent to fusing first to particle c and scatter thereafter onto the defect. We end up with the following sets of equations

$$R_a\left(\theta + i\eta_{ac}^b\right) R_b\left(\theta - i\eta_{bc}^a\right) S_{ab}(2\theta + i\eta_{ac}^b - i\eta_{bc}^a) = R_c\left(\theta\right)\,, \tag{27}$$

$$T_a\left(\theta + i\eta_{ac}^b\right) T_b\left(\theta - i\eta_{bc}^a\right) = T_c\left(\theta\right)\,, \tag{28}$$

$$T_a\left(\theta + i\eta_{ac}^b\right) R_b\left(\theta - i\eta_{bc}^a\right) S_{ab}(\theta + i\eta_{ac}^b - i\eta_{bc}^a) = 0\,, \tag{29}$$

$$R_a\left(\theta + i\eta_{ac}^b\right) T_b\left(\theta - i\eta_{bc}^a\right) = 0\,, \tag{30}$$

where we suppressed the explicit mentioning of the degrees of freedom of the impurity. Obviously we can derive the same equations also for $R \leftrightarrow \tilde{R}$ and $T \leftrightarrow \tilde{T}$. It is evident that the equations (27)-(30) only make sense when either $T = 0$ or $R = 0$, in which case the reflection bootstrap equation (27) (proposed first in [3]) and the transmission bootstrap (28) (proposed first in [24]) separately become meaningful. Hence, we have confirmed in an alternative way a statement which already followed from the previous section, namely: *A bulk theory which possesses bound states associated with a diagonal scattering matrix does not allow simultaneously non-vanishing transmission and reflection through a defect.* The argument leading to the equations (27)-(30) gives a slightly more intuitive understanding for the exclusion of this possibility, since it shows that one produces inevitably terms made up of particles on the left and on the right of the defect which can not be reconciled anymore such that (29) and (30) have to hold. Nonetheless, one should note that equation (24) is more restrictive since it also excludes theories which do not permit fusing at all, such as the sinh-Gordon model, etc.

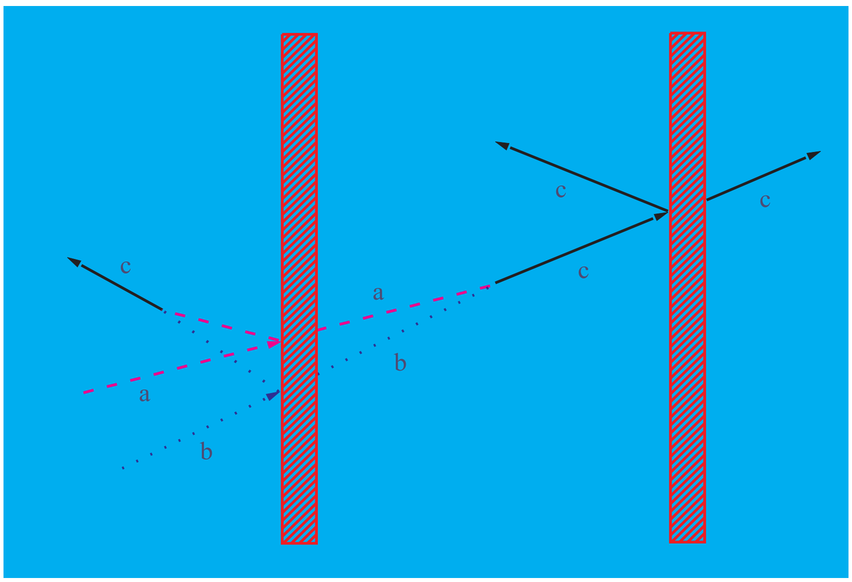

Figure 2: Defect bootstrap equations.

Let us now consider the possibility of impurity excitations and the related bootstrap equations. Having a particle of type a moving at rapidity $i\eta_{a\alpha}^{\beta}$ might change the state of the defect from α to β as

$$Z_a \left(i\eta_{a\alpha}^{\beta} \right) Z_\alpha \rightarrow Z_\beta \, . \tag{31}$$

Using this relation and commuting in accordance with the ZF-algebra (1)-(3) in an order as depicted in figure 3 leads to a set of defect bootstrap equations involving the degrees of freedom of the impurity

$$R_{b\beta}(\theta) = S_{ab} \left(\theta - i\eta_{a\alpha}^{\beta} \right) S_{ba} \left(\theta + i\eta_{a\alpha}^{\beta} \right) R_{b\alpha}(\theta) \, , \tag{32}$$

$$T_{b\beta}(\theta) = T_{b\alpha}(\theta) \, . \tag{33}$$

It follows trivially from (33) that whenever the transmission is non-vanishing there can not be any excited impurity states. In that case this is compatible with (32), since S is a constant phase which cancels due to the unitarity relation (4). On the other hand, whenever $T(\theta)$ is zero, equation (32) should be satisfied and can be used for the construction of new solutions for $R(\theta)$. The possible first order poles in $R(\theta)$, related to the fusing angles, have to be in the physical sheet, i.e., $0 < \operatorname{Im}\theta < \pi$, and should be associated with a positive residue [6]. Using these criteria one may find non-trivial closures of the boundary bound state bootstrap equation [6, 9]. Reversing this statement means, of course, that in a consistent solution for $R(\theta)$ and $T(\theta)$ every pole inside the physical sheet should be related to a negative residue. Let us verify this with explicit examples.

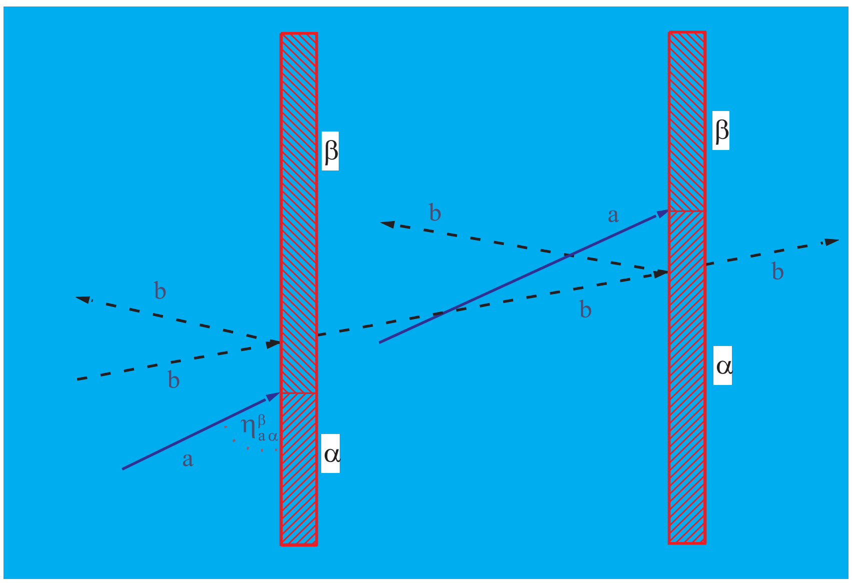

Figure 3: Defect bound state bootstrap equations.

4 The free Fermion with a defect

We consider the complex free Fermion Lagrangian density $\mathcal{L}_{FF}$ perturbed with a defect $\mathcal{D}(\bar{\psi}, \psi)$

$$\mathcal{L} = \mathcal{L}_{FF} + \delta(x)\mathcal{D}(\bar{\psi}, \psi)\,. \tag{34}$$

Here we denote as usual $\bar{\psi} = \psi^\dagger\gamma^0$, where γ^0 is one of the gamma matrices, i.e., satisfying the Clifford algebra. For the defect $\mathcal{D}(\bar{\psi}, \psi) = g\bar{\psi}\psi$ the transmission and reflection amplitudes were computed [25, 27] to be

$$R(\theta, B) \;=\; \bar{R}(\theta, -B) = -\frac{i \sin B \cosh\theta}{\sinh\theta + i\sin B}\,, \tag{35}$$

$$T(\theta,B) \;=\; \bar{T}(\theta,-B)=\frac{\cos B\sinh\theta}{\sinh\theta+i\sin B}\,. \tag{36}$$

Since Dirac Fermions are not self-conjugate, we have to distinguish particle and anti-particle. We distinguish the amplitudes related to the anti-particle by a "bar". The coupling constant g is parameterized as $\sin B=-4g/(4+g^2)$. For this example parity invariance is preserved, such that $R=\tilde{R}$ and $T=\tilde{T}$. The fact that

$$\operatorname*{Res}_{\theta\to -iB} R(\theta,B)=\operatorname*{Res}_{\theta\to -iB} T(\theta,B)=-\operatorname*{Res}_{\theta\to iB}\bar{R}(\theta,B)=-\operatorname*{Res}_{\theta\to iB}\bar{T}(\theta,B)=2\pi\sin B\,, \tag{37}$$

confirms our previous conclusion, which asserted that there can not be any excited impurity states once reflection and transmission occur simultaneously. Depending on the sign of B, the residues in (37) are either negative or the pole is beyond the physical sheet. Thus, this solution is consistent with regard to the above argumentation.

As the second example we consider the defect $\mathcal{D}(\bar{\psi},\psi)=g\bar{\psi}\gamma^0\psi$ for which the related transmission and reflection amplitudes follow from [27] as

$$R(\theta,B) \;=\; \bar{R}(\theta,B)=\frac{-i\sin B}{\sinh(\theta+iB)}\,, \tag{38}$$

$$T(\theta,B) \;=\; \bar{T}(\theta,B)=\frac{\sinh\theta}{\sinh(\theta+iB)}\,. \tag{39}$$

Also in this example parity invariance is preserved, and we have $R=\tilde{R}$ and $T=\tilde{T}$. We compute

$$\operatorname*{Res}_{\theta\to -iB} R(\theta,B)=\operatorname*{Res}_{\theta\to -iB} T(\theta,B)=2\pi\sin B\,, \tag{40}$$

such that the interpretation is the same as in the previous example and we confirm once more our general statement.

5 Conclusions

We conclude by re-stating our main results: *Upon the assumption (22) on the product in the impurity degrees of freedom, the only integrable, in the sense of factorization, bulk theories which, when doped with some impurity, allow the occurrence of simultaneous reflection and transmission are those possessing constant phases as scattering matrices. Once T and R are taken simultaneously to be non-vanishing these theories do not admit the possibility of excited impurity bound states.*

Unfortunately our main results imply that for the treatment of non-trivial impurity systems one has to leave the realm of integrable systems. Even for the small subset of theories which remain integrable in this case, the restrictive power of the integrability framework fails, since apart from crossing and unitarity we have no constraining equations at our disposal to determine the transmission and reflection amplitudes. On the other hand, this result stresses more the importance of diagonal bulk theories, whose scattering matrices are of the form (24). It would be interesting to complete the picture for these anyonic theories and seek also solutions for the Federbush type models.

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